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The a_T distribution of the Z boson at hadron colliders

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Abstract: We provide the first theoretical study of a novel variable, a_T, proposed in ref. [1] as a more accurate probe of the region of low transverse momentum p_T, for the Z boson p_T distribution at hadron colliders. The a_T is the component of p_T transverse to a suitably defined axis. Our study involves resummation of large logarithms in a_T up to the next-to-leading logarithmic accuracy and we compare the results to those for the well-known p_T distribution, identifying important physical differences between the two cases. We also test our resummed result at the two-loop level by comparing its expansion to order α_s^2 with the corresponding fixed-order results and find agreement with our expectations.

Keywords: NLO Computations, Hadronic Colliders, QCD, Standard Model

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Contents

1 Introduction

The study of W and Z boson production at hadron colliders via the Drell-Yan process [2] has formed a very significant part of particle phenomenology over a period of several years [3–5]. In the era of the LHC these studies continue to occupy an important role for a variety of reasons. For instance an accurate understanding of the production rates and transverse momentum (p_T) distributions of the W and Z, or equivalently those of lepton pairs obtained from gauge boson decays, can be used for diverse purposes. These range from more prosaic applications such as luminosity monitoring at the LHC to measurement of the W mass and perhaps most interestingly for discovery of new physics, which may manifest itself via the decay of new gauge bosons to lepton pairs.

In this paper we shall concentrate on the p_T spectrum of the Z boson and related quantities. The Z boson p_T distribution has in fact received considerable theoretical and experimental attention in the past but there remain aspects where it is desirable to have an improved understanding of certain physical issues. One such important issue is the role of the non-perturbative or "intrinsic" k_T component which has a sizable effect on the p_T spectrum at low p_T (see e.g. refs. [6–8]). The fact that the incoming quarks/anti-quarks which fuse to form gauge bosons are part of extended objects (protons or anti-protons) and have interactions with other constituents thereof generates a small transverse momentum (that can be viewed as the Fermi motion of partons inside the proton) and which a priori one might expect to be of order of the QCD scale Λ_{QCD}.

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Since the intrinsic k_T has a non-perturbative origin it cannot be computed within conventional methods of perturbative QCD. One can however model the intrinsic k_T as an essentially Gaussian smearing of perturbatively calculated p_T spectra and hope to constrain the parameters of the Gaussian by fitting the theoretical prediction to experimental data. An example of this procedure is provided by the work of Brock, Landry, Nadolsky and Yuan (BLNY) who proposed a non-perturbative Gaussian form factor that in conjunction with perturbative calculations was able to describe both Tevatron Run-1 Z data as well as lower Q^2 Drell-Yan data [8]. Alternatively one may use a Monte Carlo event generator such as HERWIG++ [9] to phenomenologically investigate the same issue. As discussed in ref. [10] these studies yield somewhat larger than expected values for the mean intrinsic k_T per parton and also reveal a dependence of this quantity on the collider energy which features are desirable to understand better.

Additionally, as pointed out by Berge et al. [11], a phenomenological study in semi-inclusive DIS processes (SIDIS) for small Bjorken-x, $x < 10^{-3}$, suggests an effective dependence of the non-perturbative form factor on x.[1] Extrapolating the effect to the LHC where such small-x values become relevant, one may expect to see significantly broader Higgs and vector boson p_T spectra than one would in the absence of small-x effects. Tevatron studies with high rapidity vector boson samples may help to provide further information on the role, if any, of small-x broadening.

Given the importance of the studies we have mentioned above, in the context of the LHC and the precise determination of p_T spectra there, it is important to have as thorough a probe of the low p_T region of Z boson production as is possible. Investigations carried out using the conventional Z boson p_T spectrum mainly suffer from large uncertainties arising from experimental systematics, dominated by resolution unfolding and the dependence on p_T of event selection efficiencies, as discussed in detail in ref. [1].

An interesting observable that is less sensitive to the above effects and also essentially insensitive to the momentum resolution of leptons produced by the decaying vector boson was proposed in ref. [1]. This variable is just the transverse component, a_T, of the p_T with respect to the lepton thrust axis, which we shall define more precisely later. It has been suggested that the a_T variable will be experimentally better determined at low a_T than the standard p_T variable and hence it would make a more accurate probe for issues such as our understanding of initial state radiation and the precise role of intrinsic k_T, including issues such as potential small-x broadening.

Before one can access information on non-perturbative effects however, it is of vital importance to have a sound perturbative estimate of the observable at hand. In the low p_T region of interest to us one is dealing with the emission of soft and/or collinear gluons which is logarithmically enhanced. The resummation of large logarithms of the form $1/p_T \left(\alpha_s^n \ln^m M^2/p_T^2\right)$, where M is the lepton-pair invariant mass and $m \leq 2n-1$, has been a subject of interest over decades [13–18] and has now been carried out to next-to-next-to leading logarithmic (NNLL) accuracy [19]. After matching such resummations with fixed-

[1]This x dependence may be merely an effective parametrisation of missing *perturbative* BFKL effects. Another observable, the vector p_T of the current hemisphere in the DIS Breit frame which can be used to investigate this x dependence, was suggested in ref. [12].

order estimates to NLO accuracy one has a state-of-the art theoretical prediction for the perturbative region.

In the present paper we address the issue of resummation for the a_T variable. We point out that there are some similarities to resummation for the p_T distribution but also some important differences that manifest themselves in the shape of the resummed distribution. We aim to provide a next-to-leading logarithmic (NLL) resummation that we envisage could be extended to NNLL level subsequently. The NLL resummed form we provide here can however already be used after matching to full next-to-leading order (NLO) results for accurate phenomenological studies of a_T.

This paper is organised along the following lines: we begin by discussing the definition of the a_T and its dependence on multiple soft gluon emission which is important at low a_T, where one encounters large logarithms. In the following section we sketch a leading-order calculation for the a_T distribution, consigning details to an appendix, which helps to illustrate some features of the a_T distribution such as the precise origin of logarithmically enhanced terms. In the subsequent section we carry out a resummation of the logarithms of a_T to NLL accuracy pointing out the relation to our recent work on azimuthal jet decorrelations [20]. Next we identify a relationship between the a_T and p_T distributions at fixed order and check this relationship with the help of a numerical fixed-order calculation using the program MCFM [21], which is a non-trivial test of our resummation. We conclude by pointing out the possibilities for further work which involve a possible extension of our resummation to NNLL accuracy (as has been done for the p_T distribution [19]) as well as matching to the MCFM results and phenomenological investigation once final experimental data becomes available.

2 Definition of $\mathbf{a_T}$ and soft limit kinematics

We will be concerned in this paper with large logarithms in the perturbative description of the a_T variable and their resummation. Since these logarithms have their origin in multiple soft and/or collinear emissions from the incoming hard partons we need to derive the dependence of the a_T on such emissions. In this section therefore we define precisely the a_T and obtain its dependence on the small transverse momenta k_t of emissions.

We recall that we are considering the production of Z bosons via the Drell-Yan (and QCD Compton) mechanisms which subsequently decay to a lepton pair. The a_T is the component of the lepton pair (or equivalently Z boson) p_T transverse to a suitably defined axis. The precise definition of the lepton thrust axis as employed in ref. [1] is provided below:

$$\hat{n} = \frac{\vec{p}_{t1} - \vec{p}_{t2}}{|\vec{p}_{t1} - \vec{p}_{t2}|}, \tag{2.1}$$

where $\vec{p}_{t1}$ and $\vec{p}_{t2}$ are the transverse momenta of the two leptons and thus $\hat{n}$ is a unit vector in the plane transverse to the beam direction. It is straightforward to verify that this is the axis with respect to which the two leptons have equal transverse momenta.

We now consider multiple emissions from the incoming partons which (neglecting the intrinsic k_T) are back-to-back along the beam direction. From conservation of transverse momentum we thus have $\vec{p}_{t1} + \vec{p}_{t2} = -\sum_i \vec{k}_{ti}$ which means that the lepton pair or Z boson p_T is just minus the vector sum of emitted gluon transverse momenta $\vec{k}_{ti}$, where we refer to the momentum transverse to the beam axis. To obtain the dependence of a_T on the k_{ti} we wish to find the component of this sum normal to the axis defined in eq. (2.1). The axis is given by (writing $\vec{p}_{t2}$ in terms of $\vec{p}_{t1}$ and $\vec{k}_{ti}$)

$$\hat{n} = \frac{2\vec{p}_{t1} + \sum_i \vec{k}_{ti}}{|2\vec{p}_{t1} + \sum_i \vec{k}_{ti}|} \approx \frac{\vec{p}_{t1}}{|\vec{p}_{t1}|}, \tag{2.2}$$

where to obtain the last equation we have neglected the dependence of the axis on emissions k_{ti}. The reason for doing so is that we are projecting the vector sum of the k_{ti} along and normal to the axis and any term $\mathcal{O}(k_{ti})$ in the definition of the axis impacts the projected quantity only at the level of terms bilinear or quadratic in the small k_{ti}. Such terms can be ignored compared to the leading linear terms $\sim k_{ti}$ that we shall retain and thus to our accuracy the axis is along the lepton direction.[2]

We can parametrise the lepton and gluon momenta in the plane transverse to the beam as below:

$$\begin{aligned} \vec{p}_{t1} &= p_t\,(1,0)\,, \\ \vec{k}_{ti} &= k_{ti}\,(\cos\phi_i, \sin\phi_i)\,, \end{aligned} \tag{2.3}$$

where ϕ_i denotes the angle made by the i^{th} emission with respect to the direction of lepton 1 in the transverse plane. It is thus clear that, expressed in these terms, the transverse component of the Z boson p_T is simply $-\sum_i k_{ti}\sin\phi_i$ and one has

$$a_T = \left|\sum_i k_{ti}\sin\phi_i\right|\,. \tag{2.4}$$

We note immediately that the dependence on soft emissions is identical to the case of azimuthal angle $\Delta\phi$ between final state dijets near the back-to-back region $\Delta\phi \approx \pi$, for which resummation was carried out in ref. [20]. This is not surprising since the component of the Z boson p_T, transverse to the axis defined above, is proportional to $\pi - \Delta\phi$, where $\Delta\phi$ is the angle between the leptons in the plane transverse to the beam. The other (longitudinal) component of Z boson p_T, a_L, is proportional in the soft limit to $p_{t1} - p_{t2}$ the difference in lepton transverse momenta.[3] This kinematics is summarised in figure 1, which shows final state momenta in the transverse plane. Together with $\vec{p}_{t1}$ and $\vec{p}_{t2}$, the two lepton transverse momenta, we have displayed the vector boson transverse momentum $\vec{p}_T$, the axis $\hat{n}$ defined in eq. (2.1), and the two transverse momentum components $\vec{a}_L$ and $\vec{a}_T$. From the figure it is also clear that the angle $\pi - \Delta\phi$, also indicated, is well approximated by $|\vec{a}_T|/|\vec{p}_{t2}| \approx a_T/p_{t1}$. In the case of dijet production the kinematics is the same, with $\vec{p}_{t1}$ and $\vec{p}_{t2}$ representing the transverse momenta of the two highest-p_t jets.

[2] To be more precise the recoil of the axis against soft emissions, if retained, corrects our result only by terms that vanish as $a_T \to 0$. Such terms are beyond the scope of NLL resummation but will be included up to NLO due to the matching.

[3] For the case of dijet production this p_t imbalance has also been addressed via resummation in ref. [22] which to our knowledge is the first extension of the p_T resummation formalism to observables involving final state jets.

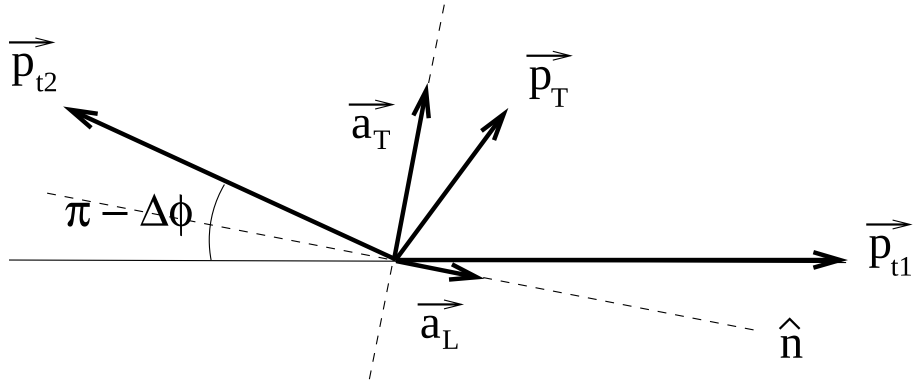

Figure 1. A di-lepton event in the plane transverse to the beam. Besides final state momenta, the two components of vector boson transverse momentum $\vec{a}_T$ and $\vec{a}_T$ are displayed, as well as $\pi - \Delta\phi$, where $\Delta\phi$ is the angle between the transverse momenta of the two leptons. See text for a full description.

Since it is possible to measure more accurately the lepton angular separation compared to their p_t imbalance (where momentum resolution is an issue), one can obtain more accurate measurements of a_T as compared to a_L or the Z boson p_T which is given by $\sqrt{a_T^2 + a_L^2}$ [1]. The resummation that we carry out here will be similar in several details to those of refs. [20, 22] but simpler since the final state hard particles are colourless leptons. We carry out a leading order calculation of the observable in question in the following section and finally the resummation.

3 Leading order result

Here we shall mention how to compute the logarithmically enhanced terms in a_T at leading order in α_s, with details of the derivation left to the appendix A. We shall highlight the origin of the double and single logarithms and relate them to the corresponding logarithms in the standard p_T distribution at the same order. The discussion here should facilitate an understanding of the resummation we carry out in the next section and the results of subsequent sections.

At Born level we have to consider the process $p_1 + p_2 = l_1 + l_2$ where p_1, p_2 and l_1, l_2 are the four momenta of incoming partons and outgoing leptons respectively. The squared matrix element for the Drell-Yan process for lepton pair production via Z decay is [23]

$$\mathcal{M}^2_{\rm DY}(l_1, l_2) = \frac{8}{N_c}\,\mathcal{G}\left(\alpha, \theta_W, M^2, M_Z^2\right)\left[A_l\, A_q\left(t_1^2 + t_2^2\right) + B_l\, B_q\left(t_1^2 - t_2^2\right)\right], \tag{3.1}$$

with the precise forms of the electroweak constant coefficients A_l, A_q and B_l, B_q as well as $\mathcal{G}$ reported in eq. (B.3). Henceforth we shall suppress the dependence of $\mathcal{G}$, which has dimension M^{-4}, on the standard electroweak parameters α, θ_W, M_Z. The factor $1/N_c$ comes from the average over initial state colours.

We have also defined the invariants[4]

$$t_1 = -2p_1.l_1 \qquad\qquad t_2 = -2p_2.l_1\,, \tag{3.2}$$

while M^2 is the invariant mass of the lepton pair which we fix. The component $t_1^2 + t_2^2$ is the parity conserving piece also present in the case of the virtual photon process while the $t_1^2 - t_2^2$ component is related to the parity violating piece of the electroweak coupling and hence absent for the photon case.

We shall study the integrated cross-section which is directly related to the number of events below some fixed value of a_T

$$\Sigma(a_T) = \int_0^{a_T} \frac{d^2\sigma}{da'_T dM^2}\, da'_T\,, \tag{3.3}$$

from which the distribution in a_T can be obtained by differentiation and the dependence of Σ on M^2 will be henceforth implied.

At the Born level the p_T of the lepton pair and hence the a_T vanishes so that the full Born contribution, evaluated at fixed mass M^2 contributes to the cross-section in eq. (3.3). Evaluating this quantity is straightforward and further explanation is available in appendix A. The result we obtain is

$$\Sigma^{(0)}(a_T) = \mathcal{G}\frac{M^2}{3\pi}\frac{A_l\,A_q}{N_c}\int_0^1 dx_1\int_0^1 dx_2\,\left[f_q(x_1)f_{\bar q}(x_2) + q\leftrightarrow \bar q\right]\,\delta\left(M^2 - s x_1 x_2\right) \equiv \Sigma^{(0)}\,, \tag{3.4}$$

where $f_q(x_1)$ and $f_{\bar q}(x_2)$ denote parton distribution functions and x_1, x_2 the momentum fractions carried by the incoming partons. We also do not explicitly indicate a sum over quark/anti-quark flavours q which should be understood. In writing the above result we have for simplicity integrated inclusively over the lepton rapidities, which results in the fact that the parity violating contribution proportional to $B_l\,B_q$ averages to zero. It is straightforward to adapt our results to include for instance acceptance cuts when final experimental data becomes available.

We now derive the QCD corrections to leading order in α_s with the aim of identifying logarithmically enhanced terms in a_T to the integrated cross-section defined in eq. (3.3). To this end we need to consider the process $p_1 + p_2 = l_1 + l_2 + k$ where k is a final state parton emission as well as $\mathcal{O}\left(\alpha_s\right)$ virtual corrections to the Drell-Yan process.

Let us focus first on the real emission contribution. We need to compute the quantity

$$\begin{aligned}\Sigma^{(1)}(a_T) = \int_0^1 dx_1\int_0^1 dx_2\,\Big\{&\left[f_q(x_1)f_{\bar q}(x_2)+q\leftrightarrow\bar q\right]\,\hat\Sigma_A^{(1)}(a_T)\\ &+\left[\left(f_q(x_1)+f_{\bar q}(x_1)\right)f_g(x_2)+q,\bar q\leftrightarrow g\right]\hat\Sigma_C^{(1)}(a_T)\Big\}\,,\end{aligned} \tag{3.5}$$

where the partonic quantities $\hat\Sigma^{(1)}_{A/C}$ which give the $\mathcal{O}(\alpha_s)$ contribution read

$$\hat\Sigma_i^{(1)}(a_T) = \int d\Phi(l_1,l_2,k)\,\mathcal{M}_i^2(l_1,l_2,k)\delta\left(M^2 - 2l_1.l_2\right)\Theta\left(a_T - k_t|\sin\phi|\right)\,, \tag{3.6}$$

[4] The quantities t_1 and t_2 were labelled as $\hat t_1$, $\hat t_2$ while l_1 and l_2 were labelled k_1 and k_2 in ref. [23].

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where the index i runs over the contributing subprocesses at this order, i.e. $i = A/C$ denotes the annihilation (Drell-Yan)/Compton subprocesses while $\mathcal{M}_i^2$ is the appropriate squared matrix element the explicit form of which we include in appendix B. We also need to carry out the integration over the three-body Lorentz invariant phase space Φ, since in addition to the final state lepton four-momenta l_1, l_2 we also have a final state emitted parton k. We have introduced a delta function constraint that indicates we are working at fixed invariant mass of the lepton pair $2l_1.l_2 = M^2$. Additionally in order to compute the integrated a_T cross-section eq. (3.3), we need to restrict the additional parton emission k such that we are studying events below some value of a_T. Recalling, from the previous section, that the value of this quantity generated by a gluon with transverse momentum k_t and angle with the lepton axis ϕ is $k_t|\sin\phi|$ we arrive at the step function in the above equation.[5] We then fold the parton level result with parton distribution functions precisely as for the Born level result $\Sigma^{(0)}$ reported above.

After integrating over all lepton variables, accounting for virtual corrections and retaining only singular terms in the limit $k_t \to 0$ (which are the source of logarithms in a_T), as detailed in appendix A, we arrive at the result for the annihilation contribution

$$\begin{aligned}\Sigma_A^{(1)}(a_T) = &-\mathcal{G}\frac{M^2}{3\pi}\frac{A_l A_q}{N_c}\int_0^1 \frac{d\Delta}{\Delta}\int_0^{1-2\sqrt{\Delta}} dz \; \times \\ &\times \int_0^1 dx_1 \int_0^1 dx_2 \left[f_q(x_1) f_{\bar{q}}(x_2) + q \leftrightarrow \bar{q}\right] \delta\left(M^2 - s x_1 x_2 z\right) \times \\ &\times \int_0^{2\pi}\frac{d\phi}{2\pi} C_F \frac{\alpha_s}{2\pi}\frac{2\left(1+z^2\right)}{\sqrt{(1-z)^2 - 4z\Delta}}\Theta\left(\sqrt{\Delta}|\sin\phi| - \frac{a_T}{M}\right),\end{aligned} \tag{3.7}$$

while that for the Compton subprocess reads

$$\begin{aligned}\Sigma_A^{(1)}(a_T) = &-\mathcal{G}\frac{M^2}{3\pi}\frac{A_l A_q}{N_c}\int_0^1 \frac{d\Delta}{\Delta}\int_0^{1-2\sqrt{\Delta}} dz \; \times \\ &\times \int_0^1 dx_1 \int_0^1 dx_2 \left[f_q(x_1) f_{\bar{q}}(x_2) + q \leftrightarrow \bar{q}\right] \delta\left(M^2 - s x_1 x_2 z\right) \times \\ &\times \int_0^{2\pi}\frac{d\phi}{2\pi} C_F \frac{\alpha_s}{2\pi}\frac{2\left(1+z^2\right)}{\sqrt{(1-z)^2 - 4z\Delta}}\Theta\left(\sqrt{\Delta}|\sin\phi| - \frac{a_T}{M}\right),\end{aligned} \tag{3.8}$$

Note that the above equations involve the step function constraint $\Theta\left(k_t|\sin\phi| - a_T\right)$ which represents the fact that the number of events with $k_t|\sin\phi| < a_T$ is equal to the total rate minus the events with $k_t|\sin\phi| > a_T$. Since the total rate is a number independent of a_T, we can simply compute the events with $k_t|\sin\phi| > a_T$ to obtain the logarithmic a_T dependence, which is what we have done above.

In the above equations we have also parametrised the integral over the gluon momentum via the rescaled transverse momentum $\Delta = k_t^2/M^2$, the azimuthal angle ϕ and z where in the collinear limit $1-z$ is just the fraction of the parent partons energy carried off by the radiated gluon. We also have as usual $C_F = 4/3$, $T_R = 1/2$, $\alpha_s = g^2/4\pi$.

[5]As we stated previously this approximation is sufficient up to terms that vanish as $a_T \to 0$, which we do not compute here.

The above results are sufficient to obtain the logarithmic structure in a_T and compare it to the corresponding result for the Z boson p_T distribution. In this respect we note that the only difference between the results reported immediately above and those for the p_T case are the $|\sin\phi|$ terms in the step function constraints above. While at the leading order these will essentially just be a matter of detail we shall see that the $\sin\phi$ dependence has an important role to play in the shape of the resummed spectrum.

To complete the calculations one proceeds as in the Z boson p_T case and hence we take the moments with respect to the standard Drell-Yan variable $\tau = \frac{M^2}{s}$, thereby defining

$$\tilde{\Sigma}(N, a_T) = \int_0^1 d\tau\, \tau^{N-1}\, \Sigma(a_T), \tag{3.9}$$

which can be expressed as a sum over the moment space annihilation and Compton terms $\tilde{\Sigma}(N, a_T) = \tilde{\Sigma}_A(N, a_T) + \tilde{\Sigma}_C(N, a_T)$.

The Born level Drell-Yan contribution can then be expressed in moment space as

$$\tilde{\Sigma}^{(0)}(N) = \frac{\mathcal{G}}{3\pi}\frac{A_l A_q}{N_c}\, F_A(N)\,, \tag{3.10}$$

where $F_A(N)$ denotes the moment integrals of the parton distribution functions

$$\begin{aligned} F_A(N) &= \int_0^1 dx_1\, x_1^N \int_0^1 dx_2\, x_2^N\, \left[f_q(x_1)\, f_{\bar{q}}(x_2) + q \leftrightarrow \bar{q}\right] \\ &= \tilde{f}_q(N)\, \tilde{f}_{\bar{q}}(N) + q \leftrightarrow \bar{q}\,, \end{aligned} \tag{3.11}$$

where we introduced $\tilde{f}(N)$, the moments of the parton distributions.

Likewise the $\mathcal{O}(\alpha_s)$ annihilation contribution can be expressed as

$$\begin{aligned} \tilde{\Sigma}_A^{(1)}(N, a_T) &= -\frac{\mathcal{G}}{3\pi}\frac{A_l A_q}{N_c}\, F_A(N) \int_0^1 \frac{d\Delta}{\Delta}\int_0^{1-2\sqrt{\Delta}} dz\, z^N \times \\ &\times \int_0^{2\pi}\frac{d\phi}{2\pi}\, C_F\frac{\alpha_s}{2\pi}\frac{2\left(1+z^2\right)}{\sqrt{(1-z)^2 - 4z\Delta}}\Theta\left(\sqrt{\Delta}|\sin\phi| - \epsilon\right)\,, \end{aligned} \tag{3.12}$$

where $\epsilon = a_T/M$ is a dimensionless version of the a_T variable.

Performing the integrals over z and Δ we obtain the result

$$\begin{aligned} \tilde{\Sigma}_A^{(1)}(N, a_T) &= -\tilde{\Sigma}^{(0)}(N)\left[2\frac{\alpha_s}{\pi}\gamma_{qq}(N)\int_0^{2\pi}\frac{d\phi}{2\pi}\ln\frac{|\sin\phi|}{\epsilon}\right. \\ &\left. + \frac{2C_F\alpha_s}{\pi}\int_0^{2\pi}\frac{d\phi}{2\pi}\left(\ln^2\frac{|\sin\phi|}{\epsilon} - \frac{3}{2}\ln\frac{|\sin\phi|}{\epsilon}\right)\right]\,. \end{aligned} \tag{3.13}$$

where we introduced the quark anomalous dimension

$$\gamma_{qq}(N) = C_F\int_0^1 dz\left(z^N - 1\right)\frac{1+z^2}{1-z}\,. \tag{3.14}$$

Notice the proportionality of the above result to the Born level result which is a consequence of the collinear origin of logarithmic terms.

We have not integrated over the variable ϕ as yet in order to make the link to results for the p_T distribution. To obtain the $\mathcal{O}(\alpha_s)$ integrated cross-section for the p_T case the same formulae as reported above apply but one replaces $|\sin\phi|$ by unity while ϵ would denote p_T/M. The ϕ integral is then trivial and can be replaced by unity. For the a_T variable on performing the ϕ integral we use the results

$$\int_0^{2\pi} \frac{d\phi}{2\pi} \ln^2|\sin\phi| = \ln^2 2 + \frac{\pi^2}{12}, \tag{3.15}$$

$$\int_0^{2\pi} \frac{d\phi}{2\pi} \ln|\sin\phi| = -\ln 2, \tag{3.16}$$

to obtain

$$\tilde{\Sigma}_A^{(1)}(N, a_T) = -\tilde{\Sigma}^{(0)}(N) \times \\ \times \left[2\frac{\alpha_s}{\pi}\gamma_{qq}(N)\ln\frac{1}{2\epsilon} + \frac{2C_F\alpha_s}{\pi}\left(\ln^2\frac{1}{2\epsilon} - \frac{3}{2}\ln\frac{1}{2\epsilon}\right) + \frac{C_F\alpha_s}{2\pi}\frac{\pi^2}{3}\right]. \tag{3.17}$$

The corresponding result for the QCD Compton process is purely single logarithmic and reads

$$\begin{aligned}\tilde{\Sigma}_C^{(1)}(N, a_T) &= -\frac{\mathcal{G}}{3\pi}\frac{A_l A_q}{N_c} F_C(N)\left(2\frac{\alpha_s}{\pi}\gamma_{qg}(N)\int_0^{2\pi}\frac{d\phi}{2\pi}\ln\frac{|\sin\phi|}{\epsilon}\right) \\ &= -\frac{\mathcal{G}}{3\pi}\frac{A_l A_q}{N_c} F_C(N)\, 2\frac{\alpha_s}{\pi}\gamma_{qg}(N)\ln\frac{1}{2\epsilon},\end{aligned} \tag{3.18}$$

where

$$\gamma_{qg}(N) = T_R\int_0^1 dz\, z^N\left[z^2 + (1-z)^2\right], \tag{3.19}$$

and $F_C(N)$ is the moment integral of the relevant combination of parton density functions

$$\begin{aligned}F_C(N) &= \int_0^1 dx_1\, x_1^N \int_0^1 dx_2\, x_2^N\, \left[(f_q(x_1) + f_{\bar{q}}(x_1))\, f_g(x_2) + q, \bar{q} \leftrightarrow g\right] \\ &= \left(\tilde{f}_q(N) + \tilde{f}_{\bar{q}}(N)\right)\tilde{f}_g(N) + q, \bar{q} \leftrightarrow g\,.\end{aligned} \tag{3.20}$$

In our final results, eqs. (3.17) and (3.18), we have neglected constant terms that are identical to those for the Drell-Yan p_T distribution computed for instance in [24].

We note that the logarithms found here, both in the Drell-Yan and Compton contributions, are the same as those for the p_T variable with the replacement $\epsilon \to 2\epsilon$. In other words as far as the logarithmic dependence is concerned we obtain that the result for the cross-section for events with $a_T < \epsilon M$ is the same as the result for the variable $p_T/2 < \epsilon M$. The only other effect, at this order, of the $|\sin\phi|$ term is to generate a constant term $\frac{C_F\alpha_s}{2\pi}\frac{\pi^2}{3}$ reported above. Thus to leading order in α_s we have simply

$$\Sigma^{(1)}(a_T)|_{a_T=\epsilon M} - \Sigma^{(1)}\left(\frac{p_T}{2}\right)|_{p_T/2=\epsilon M} = -\Sigma^{(0)}\, C_F\frac{\alpha_s}{2\pi}\frac{\pi^2}{3}. \tag{3.21}$$

In writing the above we have returned to τ space by inverting the Mellin transform so as to obtain the result in terms of the factor $\Sigma^{(0)}$ rather than $\tilde{\Sigma}^{(0)}(N)$.

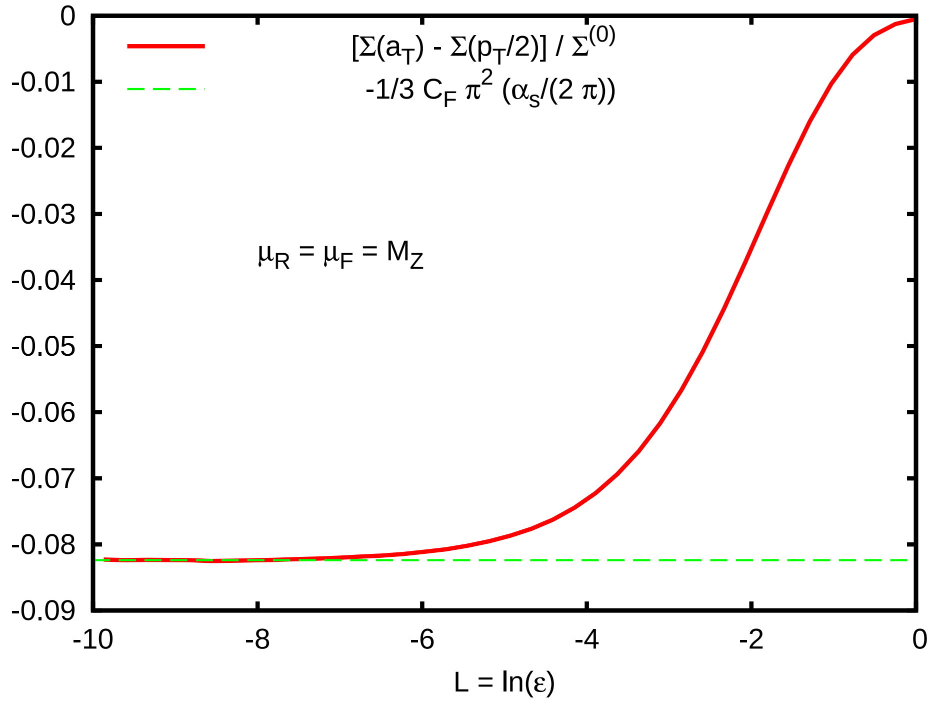

Figure 2. The difference between the integrated distributions for a_T and $p_T/2$. Here we have used the CTEQ6M pdf set [25] and both factorisation and renormalisation scales μ_F and μ_R have been fixed at the Z boson mass M_Z.

The result above can be verified by using a fixed-order program such as MCFM. One can obtain the results for the integrated cross-sections for a_T and $p_T/2$ and the difference between them should be a constant with the value reported above. This is indeed the case, as one can see from the plot in figure 2, where the difference in eq. (3.21) generated using the numerical fixed-order program MCFM [21], divided by the Born cross section $\Sigma^{(0)}$, is plotted against $L = \ln(\epsilon)$. The results from MCFM agree with our expectation (3.21). In order to show the smoothest curve we have taken the case where the Z decay has been treated fully inclusively (i.e. we have not placed rapidity cuts) and a narrow width approximation eventually employed but we have checked our results agree with MCFM for arbitrary cuts on lepton rapidities. Having carried out the fixed-order computation, which serves to illustrate some important points, we shall shift our attention to the resummation of logarithms to all orders.

4 Resummed results

Here we shall carry out the resummation of the large logarithms in the ratio of two scales M and a_T which become disparate at small a_T, $a_T \ll M$. We already derived the dependence of the a_T on multiple soft and/or collinear emissions in the preceding section and hence in order to carry out the resummation we next need to address the dynamics of multiple low

k_t emissions. We shall first treat only the Drell-Yan process and later specify the role of the QCD Compton production process.

We shall study as before the integrated cross-section representing the number of events below some fixed value of a_T, defined in eq. (3.3), from which one can obtain the a_T distribution by differentiating with respect to a_T. Also as we emphasised in the previous section we are working at fixed invariant mass of the lepton pair purely as an illustrative example and we can straightforwardly adapt our calculations to take into account experimental cuts on for instance lepton rapidities, which in any case do not affect the resummation.

We consider again the incoming partons as carrying momentum fractions x_1 and x_2 of the incoming hadrons which means that at Born level where they annihilate to form the lepton pair via virtual Z production we have simply $M^2 = \hat{s} = s x_1 x_2$, where the Mandelstam invariant $\hat{s}$ denotes the partonic centre of mass energy. Beyond the Born level one has to take account of gluon radiation and to this end we introduce as in the previous section the quantity $z = M^2/\hat{s}$ such that $1-z$ represents the fractional energy loss of the incoming partons due to the radiation of collinear gluons. Thus in the limit $z \to 1$ one is probing soft and collinear radiation while away from $z = 1$ we will be dealing with energetic collinear emission. We note here that for the purpose of generating the logarithms we resum we do not have to examine large-angle radiation and the collinear limit is sufficient as for the usual p_T distribution. In fact since the a_T resummation we aim to carry out shares several common features with the well-known p_T distribution we shall only sketch the resummation concentrating instead on features of the a_T which lead to differences from the p_T variable. For a recent detailed justification of the approximations that lead to NLL resummation for the p_T case as well as for other variables the reader is referred to ref. [26].

We work in the centre-of-mass frame of the colliding partons and in moment space where we take moments with respect to $\tau = M^2/s$ of the cross-section in eq. (3.3) as in the fixed-order calculations we carried out. Taking moments enables us to write for the emission of multiple collinear and optionally soft gluons

$$\tilde{\Sigma}(N, a_T) = \tilde{\Sigma}^{(0)}(N)\, W_N(a_T)\,, \tag{4.1}$$

where $\tilde{\Sigma}^{(0)}(N)$ is the Born level result in eq. (3.10). The effects of multiple collinear (and optionally soft) gluon emission from the incoming projectiles are included in the function W_N which can be expressed to next-to-leading logarithmic (NLL) in the standard factorised form

$$\begin{aligned} W_N^{\rm real}(a_T) = & \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=1}^{n} \int dz_i \frac{dk_{t_i}^2}{k_{ti}^2} \frac{d\phi_i}{2\pi} \times \\ & \times z_i^N \, 2C_F \frac{\alpha_s(k_{ti}^2)}{2\pi} \left(\frac{1+z_i^2}{1-z_i}\right) \Theta\left(a_T - \left|\sum_i v(k_i)\right|\right)\,, \end{aligned} \tag{4.2}$$

where $1-z_i$ denotes the fraction of momentum carried away by emission of a quasi-collinear gluon i from the incoming hard projectile so that $M^2/\hat{s} = z = \prod_i z_i$ and k_{ti} is the transverse momentum of gluon i with respect to the hard emitting incoming partons.

In writing the above results we have used an independent emission approximation, valid to NLL accuracy, where the emission probability for n collinear gluons is merely the product of single gluon emission probabilities, which factorise from the Born level production of the hard lepton pair.[6] The single gluon emission probability to the same NLL accuracy is given by the leading order splitting function for the splitting of a quark to a quasi-collinear quark and gluon (weighted by the running strong coupling),

$$P_{qq}(z)\frac{\alpha_s(k_t^2)}{2\pi} = C_F\frac{\alpha_s(k_t^2)}{2\pi}\frac{1+z^2}{1-z}\,, \tag{4.3}$$

with α_s defined in the CMW scheme [29]. We have inserted a factor of two to take account of the fact that there are two hard incoming partons which independently emit collinear gluons. We have also taken care of the constraint on real gluon emission, imposed by the requirement that the sum of the components of the k_{ti} normal to the axis in eq. (2.1) (denoted by $v(k_i) = k_{ti}\sin\phi_i$) is less than a_T. We have integrated over the leptons, holding the invariant mass M fixed, and taken moments to obtain the full zeroth order Drell-Yan result $\tilde{\Sigma}^{(0)}(N)$, which multiplies the function W_N containing all-order radiative effects.

All of the above arguments would also apply to the case of the p_T variable. Thus while the dynamics of multiple soft/collinear emission is treated exactly as for the p_T resummation the difference between the p_T and our resummation arises purely due to the different form of the argument of the step function restricting multiple real emission. Thus while for the p_T variable the phase space constraint involves a two-dimensional vector sum $\Theta\left(p_T - |\sum_i \vec{k_{ti}}|\right)$, in the present case we have a one dimensional sum of the components of the gluon k_t normal to the lepton thrust axis, $v(k_i) = k_{ti}\sin\phi_i$. One encounters such a one dimensional sum also in cases such as azimuthal correlations in DIS [20, 30] and the resummation of the p_t difference between jets in dijet production [22]. It is this difference that will be responsible for different features of the a_T distribution as we shall further clarify below. The relationship between azimuthal correlations and the a_T is no surprise since the a_T variable is proportional to $\pi - \Delta\phi_{ll}$, the deviation of the azimuthal interval between the leptons from its Born value π.

In order to further simplify eq. (4.2) we also factorise the phase space constraint using a Fourier representation of the step function [30]

$$\Theta\left(a_T - |\sum_i v(k_i)|\right) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{db}{b}\sin(ba_T)\prod_i e^{ibv(k_i)}. \tag{4.4}$$

Note the presence of the $\sin(ba_T)$ function which is a consequence of addressing a one dimensional sum as opposed to the Bessel function J_1 one encounters in resummation of the p_T cross-section. With both the multiple emission probability and phase space factorised as above it is easy to carry out the infinite sum in eq. (4.2) which yields

$$W_N^{\text{real}}(a_T) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{db}{b}\sin(ba_T)\,e^{R^{\text{real}}(b)}\,, \tag{4.5}$$

[6] This approximation is invalid for situations when one is examining soft radiation in a limited angular interval away from hard emitting particles [27, 28], which is not the case here.

with the exponentiated real gluon emission contribution

$$R^{\mathrm{real}(b)} = \int dz \frac{dk_t^2}{k_t^2} \frac{d\phi}{2\pi} \, z^N \, 2\, C_F \frac{\alpha_s(k_t^2)}{2\pi} \left(\frac{1+z^2}{1-z}\right) e^{ibv(k)} \, \Theta\left(1 - z - \frac{k_t}{M}\right) . \tag{4.6}$$

The kinematic limit on the z integration is set in such a way that one correctly accounts for soft large angle emissions.

Next we include all-order virtual corrections which straightforwardly exponentiate in the soft-collinear limit to yield finally

$$W_N(a_T) = \frac{2}{\pi} \int_0^\infty \frac{db}{b} \sin(b\, a_T)\, e^{-R(b)} \,, \tag{4.7}$$

where

$$\begin{aligned} -R(b) = R^{\mathrm{real}} + R^{\mathrm{virtual}} &= \int dz \frac{dk_t^2}{k_t^2} \frac{d\phi}{2\pi} 2C_F \frac{\alpha_s(k_t^2)}{2\pi} \left(\frac{1+z^2}{1-z}\right) \times \\ &\times \left(z^N \, e^{ibv(k)} - 1\right) \Theta\left(1 - z - \frac{k_t}{M}\right), \end{aligned} \tag{4.8}$$

where it should be clear that the term corresponding to the -1 added to the real contribution $z^N e^{ibv(k)}$ corresponds to the virtual corrections. Note that the virtual corrections are naturally independent of both Fourier and Mellin variables b and N respectively since they do not change the longitudinal or transverse momentum of the incoming partons and hence exponentiate directly. We are thus left to analyse $R(b)$ the "radiator" up to single-logarithmic accuracy.

4.1 The resummed exponent

Here we shall evaluate the function $R(b)$ representing the resummed exponent to the required accuracy. We shall first explicitly introduce a factorisation scale Q_0^2 to render the integrals over k_t finite. Later we will be able to take the $Q_0 \to 0$ limit. Thus one considers all emissions with transverse momenta below Q_0 to be included in the pdfs which are defined at scale Q_0 such that the factor $\tilde{\Sigma}^{(0)}(N)$ reads

$$\tilde{\Sigma}^{(0)}(N) = \frac{A_l A_q}{N_c} \frac{\mathcal{G}}{3\pi} F_A(N, Q_0^2) \,, \tag{4.9}$$

with

$$\begin{aligned} F_A(N, Q_0^2) &= \int_0^1 dx_1 x_1^N \int_0^1 dx_2 x_2^N \left[f_q(x_1, Q_0^2) f_{\bar{q}}(x_2, Q_0^2) + q \leftrightarrow \bar{q}\right] \\ &= \left[\tilde{f}_q(N, Q_0^2)\, \tilde{f}_{\bar{q}}(N, Q_0^2) + q \leftrightarrow \bar{q}\right] . \end{aligned} \tag{4.10}$$

Thus while in the fixed-order calculation of the previous section the pdfs could be treated as bare scale independent quantities for the resummed calculation we start with full pdfs evaluated at an arbitrary (perturbative) factorisation scale. The k_t integration in the perturbative radiator should now be performed with scale $k_t > Q_0$. We next follow the method of ref. [31] where an essentially identical integral was performed for the radiator.

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First, following the method of ref. [31] we change the argument of the pdfs from Q_0 to the correct hard scale of the problem, the pair invariant mass M, via DGLAP evolution. To be precise we use for the quark distribution

$$\tilde{f}_q(N,Q_0^2) = \tilde{f}_{\bar{q}}(N,M^2)\, e^{-\int_{Q_0^2}^{M^2} \frac{dk_t^2}{k_t^2}\frac{\alpha_s(k_t^2)}{2\pi}\gamma_{qq}(N)}\,, \tag{4.11}$$

and likewise for the anti-quark distribution where $\gamma_{qq}(N)$ is the standard quark anomalous dimension matrix. Note that we have not yet considered the QCD Compton scattering process and the corresponding evolution of the quark pdf from incoming gluons via the γ_{qg} anomalous dimension matrix, which we shall include in the final result by using the full pdf evolution rather than the simplified form reported immediately above. Carrying out the above step results in a modified radiator such that one now has

$$\begin{aligned} R(b) \,=\, & 2C_F \int_{Q_0^2}^{M^2} \frac{dk_t^2}{k_t^2}\frac{\alpha_s(k_t^2)}{2\pi}\int_0^{2\pi}\frac{d\phi}{2\pi}\,\times \\ & \times \left[\int_0^1 dz\,\frac{1+z^2}{1-z}\left(1-z^N e^{ibv(k)}\right)\Theta\left(1-z-\frac{k_t}{M}\right)+\frac{\gamma_{qq}(N)}{C_F}\right]. \end{aligned} \tag{4.12}$$

Using the definition of the anomalous dimension $\gamma_{qq}(N)$ we can write the above (see for instance ref. [31]) as

$$R(b) = 2C_F \int_{Q_0^2}^{M^2} \frac{dk_t^2}{k_t^2}\frac{\alpha_s(k_t^2)}{2\pi}\int_0^{2\pi}\frac{d\phi}{2\pi}\int_0^1 dz\, z^N\frac{1+z^2}{1-z}\left(1-e^{ibv(k)}\right)\Theta\left(1-z-\frac{k_t}{M}\right), \tag{4.13}$$

where in arriving at the last equation we neglected terms of $\mathcal{O}\left(k_t/M\right)$.

To NLL accuracy we can further make the approximation [30]

$$1-e^{ibv(k)} \approx \Theta\left(k_t|\sin\phi| - \frac{1}{\bar{b}}\right), \tag{4.14}$$

where we used the fact that in the present case $v(k_i) = k_t \sin\phi_i$ with $\bar{b} = b\, e^{\gamma_E}$. Thus one gets for the radiator $R(b) \equiv R(\bar{b})$ with

$$\begin{aligned} R(\bar{b}) \,=\, & 2C_F \int_{Q_0^2}^{M^2} \frac{dk_t^2}{k_t^2}\frac{\alpha_s(k_t^2)}{2\pi}\int_0^{2\pi}\frac{d\phi}{2\pi}\int_0^1 dz\, z^N\frac{1+z^2}{1-z}\,\times \\ & \times\Theta\left(k_t|\sin\phi|-\frac{1}{\bar{b}}\right)\Theta\left(1-z-\frac{k_t}{M}\right). \end{aligned} \tag{4.15}$$

We now evaluate the radiator to the required accuracy. Performing the z integration to NLL accuracy, and neglecting terms of relative order k_t/M, one arrives at

$$R(\bar{b}) = 2C_F\int_0^{2\pi}\frac{d\phi}{2\pi}\int_0^{M^2}\frac{dk_t^2}{k_t^2}\frac{\alpha_s(k_t^2)}{2\pi}\Theta\left(k_t|\sin\phi|-\frac{1}{\bar{b}}\right)\left(\ln\frac{M^2}{k_t^2}-\frac{3}{2}+\frac{\gamma_{qq}(N)}{C_F}\right), \tag{4.16}$$

where in the last line we took $Q_0 \to 0$ since the k_t integral is now cut-off by the step function $\Theta\left[k_t|\sin\phi| - 1/\bar{b}\right]$ and chose to perform the ϕ integration at the end. We note that the only difference between the radiator above and the standard p_T distribution is the factor $|\sin\phi|$ multiplying k_t in the step function condition above. This will result in an additional single-logarithmic contribution not present in the p_T resummation results.

In order to deal with the ϕ dependence to NLL accuracy we expand (as in ref. [22]) eq. (4.16) about $|\sin\phi| = 1$ in powers of $\ln|\sin\phi|$ to obtain

$$
\begin{aligned}
R(\bar{b}) &= 2C_F \int_0^{M^2} \frac{dk_t^2}{k_t^2} \frac{\alpha_s(k_t^2)}{2\pi} \Theta\left(k_t - \frac{1}{\bar{b}}\right)\left(\ln\frac{M^2}{k_t^2} - \frac{3}{2} + \frac{\gamma_{qq}(N)}{C_F}\right) \\
&\quad + \frac{\partial R(\bar{b})}{\partial \ln(\bar{b}M)} \int_0^{2\pi} \frac{d\phi}{2\pi} \ln|\sin\phi| + \cdots
\end{aligned} \tag{4.17}
$$

where we have neglected higher derivatives of R as they will contribute only beyond NLL accuracy. Moreover in evaluating $\partial R(\bar{b})/\partial \ln(\bar{b}M)$ we can replace R by its leading logarithmic form $R_{\rm LL}(\bar{b})$ since logarithmic derivatives of any next-to-leading logarithmic pieces of $R(\bar{b})$ will give only NNLL terms that are beyond our accuracy. The first term on the r.h.s. of the above equation is in fact just the radiator we would get for resummation of the Z boson p_T distribution which contains both leading and next-to-leading logarithmic terms. The second term on the r.h.s. accounts for the ϕ dependence of the problem and is purely next-to-leading logarithmic in nature since it contains the logarithmic derivative of $R_{\rm LL}(\bar{b})$. Thus we need to evaluate the first term on the r.h.s. of eq. (4.16) and then isolate its leading-logarithmic piece to compute the second term on the r.h.s. .

We carry out the integral over k_t to NLL accuracy using standard techniques [32], i.e. we change the coupling from the CMW to the $\overline{\rm MS}$ scheme

$$
\alpha_s^{\rm CMW}(k_t^2) = \alpha_s^{\overline{\rm MS}}(k_t^2)\left(1 + K\,\frac{\alpha_s^{\overline{\rm MS}}(k_t^2)}{2\pi}\right), \qquad K = C_A\left(\frac{67}{18} - \frac{\pi^2}{6}\right) - \frac{5}{9}n_f\,, \tag{4.18}
$$

and use a two-loop running coupling

$$
\alpha_s(k_t^2) = \frac{\alpha_s(M^2)}{1-\rho}\left[1 - \alpha_s(M^2)\frac{\beta_1}{\beta_0}\frac{\ln(1-\rho)}{1-\rho}\right], \qquad \rho = \alpha_s(M^2)\beta_0 \ln\frac{M^2}{k_t^2}\,, \tag{4.19}
$$

where the beta function coefficients are defined as

$$
\beta_0 = \frac{11C_A - 2n_f}{12\pi}, \qquad \beta_1 = \frac{17C_A^2 - 5C_A n_f - 3C_F n_f}{24\pi^2}. \tag{4.20}
$$

and $\alpha_s(M^2)$ is a shorthand for $\alpha_s^{\overline{\rm MS}}(M^2)$. We then obtain the usual expression

$$
R(\bar{b}) = L g_1(\alpha_s L) + g_2(\alpha_s L)\,, \tag{4.21}
$$

with $L \equiv \ln\left(\bar{b}^2 M^2\right)$. The functions g_1 and g_2 are then the leading and next-to-leading logarithmic functions which have the following detailed form

$$
g_1(\lambda) = \frac{C_F}{\pi\beta_0\lambda}\left[-\lambda - \ln(1-\lambda)\right], \tag{4.22}
$$

$$
\begin{aligned}
g_2(\lambda) &= \frac{3C_F}{2\pi\beta_0}\ln(1-\lambda) - \frac{\gamma_{qq}(N)}{\pi\beta_0}\ln(1-\lambda) + \frac{2C_F}{\pi\beta_0}\frac{\lambda}{1-\lambda}(-\ln 2) \\
&\quad + \frac{KC_F[\lambda + (1-\lambda)\ln(1-\lambda)]}{2\pi^2\beta_0^2(1-\lambda)} - \frac{C_F\beta_1}{\pi\beta_0^3}\left[\frac{\lambda + \ln(1-\lambda)}{1-\lambda} + \frac{1}{2}\ln^2(1-\lambda)\right],
\end{aligned} \tag{4.23}
$$

with $\lambda = \alpha_s(M^2)\,\beta_0 L$.

Let us comment on the origin of various terms. The leading logarithmic function $g_1(\lambda)$ arises from soft and collinear emission integrated over the phase space with a running coupling (to be precise the one-loop running of the coupling is sufficient to give us g_1). It is identical to the corresponding function for Z boson p_T resummation and at this level the a_T and p_T variables do not differ. The function g_2 embodies hard-collinear radiation (and hence the appearance of the quark anomalous dimension $\gamma_{qq}(N)$) as well as the two-loop running of the coupling and the change to the $\overline{\text{MS}}$ scheme from the CMW scheme which gives rise to the piece proportional to K. It is also the same as the corresponding function for Z boson p_T resummation *except* for the additional single-logarithmic term $\frac{2C_F}{\pi\beta_0}\frac{\lambda}{1-\lambda}(-\ln 2)$ which arises from the ϕ dependence of the problem. In other words one has explicitly

$$\frac{\partial R_{\rm LL}(\bar{b})}{\partial \ln(\bar{b}M)} \int_0^{2\pi} \frac{d\phi}{2\pi} \ln|\sin\phi| = \frac{\partial L g_1(\lambda)}{\partial \ln(\bar{b}M)}(-\ln 2) = \frac{2C_F}{\pi\beta_0}\frac{\lambda}{1-\lambda}(-\ln 2)\,. \tag{4.24}$$

However, this term, within NLL accuracy, can be absorbed in the radiator with a change in the definition of its argument $\bar{b}$:

$$R(\bar{b}) + \frac{\partial R_{\rm LL}(\bar{b})}{\partial \ln(\bar{b}M)} \int_0^{2\pi} \frac{d\phi}{2\pi} \ln|\sin\phi| = R(\bar{b}) - \frac{\partial R_{\rm LL}(\bar{b})}{\partial \ln(\bar{b}M)} \ln 2 \simeq R(\bar{b}/2)\,, \tag{4.25}$$

and $R(\bar{b}/2) = R(b\,e^{\gamma_E}/2)$ is precisely the radiator for the Z boson p_T distribution (see e.g. ref. [19]). As a final step we can use the anomalous dimension matrix $\gamma_{qq}(N)$ and the corresponding contribution $\gamma_{qg}(N)$ from Compton scattering which we have for brevity avoided treating, to evolve the pdfs from scale M^2 to scale $(2/\bar{b})^2$ precisely as in the case of the p_T variable. After absorption of the N dependent piece of the radiator into a change of scale of the pdfs it is trivial to invert the Mellin transform to go from N space to τ space. We can thus schematically write the result for the a_T cross-section defined in (3.3) resummed to NLL accuracy as

$$\begin{aligned}\Sigma(a_T) = {} & \mathcal{G}\frac{M^2}{3\pi}\frac{A_l A_q}{N_c}\frac{2}{\pi}\int_0^{\infty}\frac{db}{b}\sin(b a_T)\exp\left(-S(\bar{b}/2)\right) \times \\ & \times \int_0^1 dx_1 \int_0^1 dx_2 \left[f_q\left(x_1,(2/\bar{b})^2\right) f_{\bar{q}}\left(x_2,(2/\bar{b})^2\right) + q \leftrightarrow \bar{q}\right] \delta(M^2 - x_1 x_2 s)\,.\end{aligned} \tag{4.26}$$

The function $S(\bar{b}/2)$ is just the radiator $R(\bar{b})$ without the N dependent anomalous dimension terms which have been absorbed into the pdfs:

$$S(\bar{b}/2) = 2C_F \int_{(2/\bar{b})^2}^{M^2} \frac{dk_t^2}{k_t^2}\frac{\alpha_s(k_t^2)}{2\pi}\left(\ln\frac{M^2}{k_t^2} - \frac{3}{2}\right)\,. \tag{4.27}$$

As we just pointed out this function coincides with the corresponding function in the p_T case. All differences between a_T and p_T thus arise from the fact that one has to convolute the resummed exponent and pdfs with a sine function representing the constraint on a single component of k_t, rather than a Bessel function representing a constraint on both components of the k_t. Having achieved the resummation we shall next expand our resummed result to order α_s^2 and compare its expansion to fixed-order results to non-trivially test the resummation.

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5 Comparison to fixed-order results

In order to non-trivially test the resummation we have the option of expanding the results to order α_s^2 (i.e. up to two-loop corrections to the Born level) and testing the logarithmic structure against that emerging from fixed-order calculations. Since the results for the p_T distribution are already well-known and since many terms are common to the a_T and p_T resummed results it is most economical to provide a prediction for the difference between the a_T and p_T variables. To be precise we already identified a leading-order result for the difference between cross-sections involving a_T and $p_T/2$ in eq. (3.21). In this section we shall derive this difference at NLO level and compare to fixed-order estimates.

Let us consider the resummed results for the a_T and $p_T/2$ cases. We remind the reader of the well-known result for the p_T variable by expressing the integrated cross-section for events with $p_T/2$ below a fixed value ϵM:

$$\Sigma\left(p_T/2\right)|_{p_T/2=\epsilon M} = \left(1 + C_1(N)\,\frac{\alpha_s}{2\pi}\right)\tilde{\Sigma}^{(0)}(N)\int_0^{\infty} db\, 2M\epsilon\, J_1(b\, 2M\epsilon)\, e^{-R\left(\frac{\bar{b}}{2}\right)}\,, \tag{5.1}$$

The above result is expressed in moment space and we have additionally provided a multiplicative coefficient function $(1 + C_1(N)\,\alpha_s/(2\pi))$, so that the result accounts also for constant terms at leading order. This form of the resummation is correct up to NNLL accuracy in the cross-section whereas the pure resummed result without the multiplicative constant piece is correct to NLL accuracy in the resummed exponent [32]. Thus with the constant C_1 in place the resummation should guarantee at order α_s^2 terms varying as $\alpha_s^2L^4$, $\alpha_s^2L^3$ as well as the $\alpha_s^2L^2$ term which partially originates from a "cross-talk" between the $\alpha_s C_1$ term and the $\alpha_s L^2$ term in the expansion of the exponent.[7]

The equivalent result for the a_T variable is

$$\Sigma\left(a_T\right)|_{a_T=\epsilon M} = \left(1 + \bar{C}_1(N)\,\frac{\alpha_s}{2\pi}\right)\tilde{\Sigma}^{(0)}(N)\frac{2}{\pi}\int_0^{\infty}\frac{db}{b}\,\sin\left(bM\epsilon\right) e^{-R\left(\frac{\bar{b}}{2}\right)}\,, \tag{5.2}$$

where $\bar{C}_1$ is the constant for the a_T variable. We shall first expand the resummation to order α_s and consider the difference between a_T and $p_T/2$. First we express the radiator in the standard notation [32]

$$-R\left(\bar{b}/2\right) = \sum_{n=0}^{\infty}\sum_{m=0}^{n+1} G_{nm}\,\bar{\alpha}_s{}^n L^m\,, \qquad L = \ln\left(\bar{b}^2M^2/4\right)\,, \tag{5.3}$$

with $\bar{\alpha}_s = \alpha_s/2\pi$. Having done so we expand the resummed exponent so that to order α_s we can write for the p_T variable

$$\begin{aligned}\Sigma\left(p_T/2\right)|_{p_T/2=\epsilon M} &= \left(1 + C_1(N)\,\bar{\alpha}_s\right)\tilde{\Sigma}^{(0)}(N)\int_0^{\infty} db\, 2M\epsilon\, J_1(b\, 2M\epsilon)\,\times \\ &\times \left(1 + G_{11}\bar{\alpha}_s L + G_{12}\bar{\alpha}_s L^2 + \mathcal{O}(\alpha_s^2)\right)\,,\end{aligned} \tag{5.4}$$

[7] This form of the result we use is an oversimplification since we consider only the piece of the $\mathcal{O}(\alpha_s)$ constant which is associated to the annihilation channel. In principle one should also include the constant arising from the Compton channel but this is identical to the corresponding constant for the p_T variable [24] and it is straightforward to show that its effects cancel to the accuracy we need for the result we derive below for the difference between a_T and $p_T/2$ variables.

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where we replaced the resummed exponent by its expansion to order α_s.

Carrying out the b integral yields

$$\Sigma\left(p_T/2\right)|_{p_T/2=\epsilon M} = \left(1 + C_1(N)\bar{\alpha}_s\right)\tilde{\Sigma}^{(0)}(N)\left(1 + G_{11}\bar{\alpha}_s \ln\left[\frac{1}{4\epsilon^2}\right] + G_{12}\bar{\alpha}_s \ln^2\left[\frac{1}{4\epsilon^2}\right]\right), \tag{5.5}$$

where for the moment we do not insert the explicit forms of the G_{nm} coefficients.

Repeating the exercise for the a_T variable one obtains

$$\begin{aligned}\Sigma(a_T)|_{a_T=\epsilon M} &= \left(1 + \bar{C}_1(N)\bar{\alpha}_s\right)\tilde{\Sigma}^{(0)}(N) \times \\ &\times \left(1 + G_{11}\bar{\alpha}_s \ln\left[\frac{1}{4\epsilon^2}\right] + G_{12}\bar{\alpha}_s \ln^2\left[\frac{1}{4\epsilon^2}\right] + G_{12}\,\bar{\alpha}_s\,\frac{\pi^2}{3}\right),\end{aligned} \tag{5.6}$$

where we labelled the constant piece as $\bar{C}_1$ to distinguish it from that for the p_T variable. Constructing the difference at $\mathcal{O}(\alpha_s)$ between the a_T and $p_T/2$ variables we find that all the logarithms cancel and we obtain

$$\Sigma(a_T)|_{a_T=\epsilon M} - \Sigma\left(p_T/2\right)|_{p_T/2=\epsilon M} = \tilde{\Sigma}^{(0)}(N)\,\bar{\alpha}_s\left(\bar{C}_1(N) - C_1(N) + G_{12}\,\frac{\pi^2}{3}\right). \tag{5.7}$$

The value of the resummation coefficient G_{12} can be obtained from eq. (4.23) by expanding the result in powers of λ from which we find $G_{12} = -C_F$. Comparing this result with our explicit leading order calculation eq. (3.21) we find that $C_1 = \bar{C}_1$.

Next we carry out the expansion of our resummation to order α_s^2 and construct the difference from $p_T/2$ at this order. We shall then compare our expectation with MCFM. Expanding the radiator to order α_s^2 one gets

$$\begin{aligned}e^{-R\left(\frac{\bar{b}}{2}\right)} =& 1 + \bar{\alpha}_s\left(G_{11}L + G_{12}L^2\right) \\ &+ \bar{\alpha}_s^2\left(\frac{G_{11}^2 L^2}{2} + G_{22}L^2 + G_{11}G_{12}L^3 + G_{23}L^3 + \frac{G_{12}^2 L^4}{2}\right).\end{aligned} \tag{5.8}$$

Retaining only the $\mathcal{O}(\alpha_s^2)$ terms we can once again carry out the b space integrals as before and in particular the new integrals that appear at this order are

$$\begin{aligned}I_{a_T}^{(p)} &= \frac{2}{\pi}\int_0^\infty \frac{db}{b}\sin(bM\epsilon)\ln^p\left(\bar{b}^2\frac{M^2}{4}\right), \\ I_{p_T}^{(p)} &= \int_0^\infty db\, 2M\epsilon\, J_1(b\, 2M\epsilon)\ln^p\left(\bar{b}^2\frac{M^2}{4}\right),\end{aligned} \tag{5.9}$$

with $p = 3, 4$. Carrying out the above integrals with $p = 3$ is straightforward and the difference between the integrals for the p_T and a_T case with $p = 3$ produces only an $\alpha_s^2 \ln 1/\epsilon^2$ term apart from constant pieces. Such terms are beyond the accuracy of our resummation which ought to guarantee only terms as singular as $\alpha_s^2 \ln^2 1/\epsilon^2$ in the cross-section and hence to our accuracy there will be no contribution for $p = 3$ for the difference between a_T and $p_T/2$.

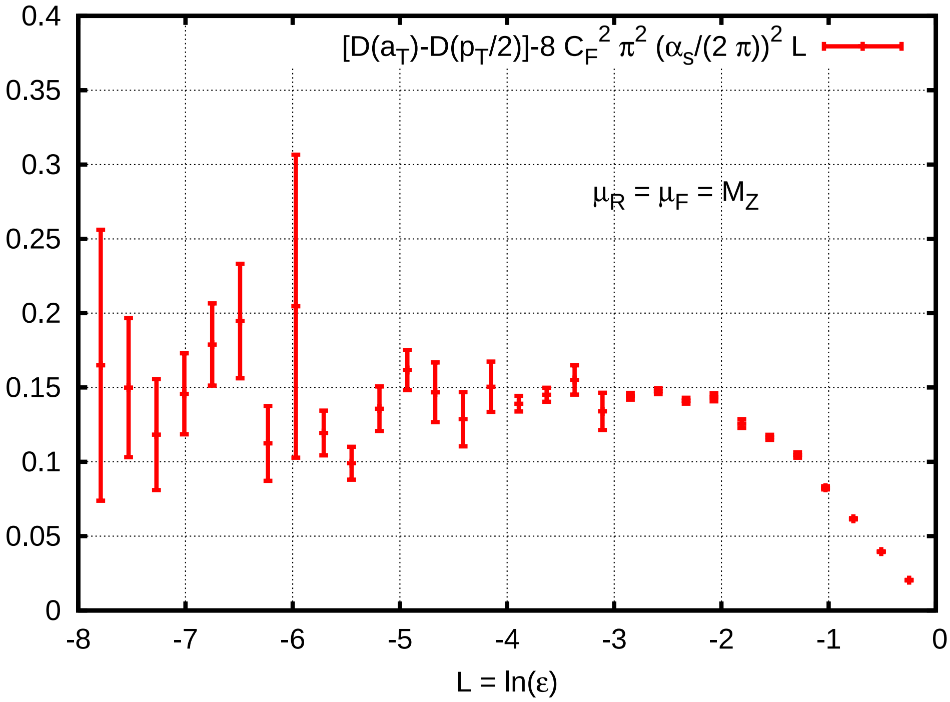

Figure 3. The difference between $D(a_T)$ and $D(p_T/2)$, defined in eq. (5.11) with the subtraction of the computed logarithmic enhanced term in eq. (5.10).

The situation changes when we consider the $p = 4$ integrals. All relevant logarithms cancel between a_T and $p_T/2$ once again except a term varying as $\alpha_s^2 \ln^2 \frac{1}{\epsilon^2}$. To be precise considering only the order α_s^2 terms one obtains

$$\begin{aligned} \Sigma(a_T)|_{a_T=\epsilon M} - \Sigma\left(p_T/2\right)|_{p_T/2=\epsilon M} &= \frac{G_{12}^2}{2} \times 2\pi^2 \bar{\alpha}_s^2 \ln^2 \frac{1}{\epsilon^2} \times \tilde{\Sigma}^{(0)}(N) + \mathcal{O}(\alpha_s^2 L) \\ &= \pi^2 \, \bar{\alpha}_s{}^2 \, C_F^2 \, \ln^2\left(\frac{1}{\epsilon^2}\right) \times \tilde{\Sigma}^{(0)}(N). \end{aligned} \tag{5.10}$$

To convert the result above back into τ space from Mellin space is straightforward as one just inverts the Mellin transform for $\tilde{\Sigma}^{(0)}$ to yield the Born-level quantity $\Sigma^{(0)}$ as the multiplicative factor.

Once again the above result can be tested against the results from MCFM. We consider the difference in the differential distributions (derivative with respect to $\ln\epsilon$ of the appropriate integrated cross-sections) for a_T and $p_T/2$ as a function of $\ln\epsilon$

$$D(a_T)|_{a_T=\epsilon M} - D\left(p_T/2\right)|_{p_T/2=\epsilon M} = \frac{1}{\Sigma^{(0)}}\left[\frac{d\Sigma(a_T)}{d\ln\epsilon}|_{a_T=\epsilon M} - \frac{d\Sigma\left(p_T/2\right)}{d\ln\epsilon}|_{p_T/2=\epsilon M}\right]. \tag{5.11}$$

Our prediction for this difference can be obtained by taking the derivative with respect to $\ln\epsilon$ of the r.h.s. of eq. (5.10). Subtracting this prediction from the MCFM results should yield at most constant terms arising from the logarithmic derivative of formally subleading $\alpha_s^2 \ln\epsilon$ terms. That this is the case can be seen from figure 3 where we note that at sufficiently small values of ϵ the difference between MCFM and our prediction tends to a constant.

6 Discussion and conclusions

Before concluding we should comment on the resummed result eq. (4.26). First we note that there is the usual issue that is involved with b space resummation of the large and small b behaviour of the integrand in that the resummed exponent diverges in both limits. The small b region is conjugate to the large k_t regime which is beyond the jurisdiction of our resummation. At sufficiently large b on the other hand we run into non-perturbative effects to do with the Landau pole in the running coupling. These issues can be resolved by modifying the radiator such that the perturbative resummation is not impacted. For instance the strategy adopted in ref. [33] was to replace the resummation variable b by another variable b^* which coincides with b in the large b limit but at small b ensures that the radiator goes smoothly to zero. Likewise to regulate the Landau pole a cut-off was placed in the large b region of integration in the vicinity of the Landau pole and it was checked that varying the position of the cut-off had no impact on the resummation. Other prescriptions can be found for instance in [34].

As far as the behaviour of the resummed cross-section and consequently the corresponding differential distribution is concerned the difference from the p_T distribution is solely due to the convolution of the resummed b space function with the $\sin(b)$ function rather than a Bessel function. As was explained in detail in ref. [30] the result of convolution with a sine function produces a distribution that does not have a Sudakov peak. The physical reason for this is that a small value of a_T can be obtained by two competing mechanisms. One mechanism is Sudakov suppression of gluon radiation and this is encapsulated to NLL accuracy by the resummed exponent. The other mechanism is the vectorial cancellation of contributions from arbitrarily hard emissions which in this case involves cancellation only of a single component of k_t transverse to the lepton axis. This mechanism is represented by the presence of the sine function while a two-dimensional constraint such as that for the p_T variable is represented by a Bessel function. In the case of one dimensional cancellation such as for the a_T as well as for instance for the dijet $\Delta\phi$ variable [20] the cancellation mechanism dominates the Sudakov suppression mechanism before the formation of the Sudakov peak while for the p_T case the vectorial cancellation sets in as the dominant mechanism after the formation of the Sudakov peak. Thus for the a_T distribution one sees no Sudakov peak but the distribution rises monotonically to a constant value as predicted by eq. (4.26).

To conclude, in this paper we have carried out a theoretical study based of a variable, a_T proposed in ref. [1] as an accurate probe of the low p_T region of the Z boson p_T distribution. Having accurate data on the a_T well into the low a_T domain will be invaluable in pinning down models of the non-perturbative intrinsic k_t and may lead to firmer conclusions on aspects such as small-x broadening of p_T distributions [8] than have been reached at present with the p_T variable. In this respect it may also be of interest to examine theoretically the power corrections to the a_T distribution along the same lines as for the p_T case [35]

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and hence to examine theoretically whether the a_T and p_T ought to have identical non-perturbative behaviour. This is once again work in progress.

Before any such conclusions can be arrived at however, it is of vital importance to have as accurate a perturbative prediction as possible to avoid misattributing missing perturbative effects to other sources. The most accurate perturbative prediction one can envisage for the a_T case is one where resummation of large logarithms in a_T is supplemented by matching to fixed-order corrections up to the two-loop level. In this paper we have carried out the first step by resumming to NLL accuracy the a_T distribution and checking our resummation by comparing to the logarithmic structure in exact fixed order calculations. We envisage that it should be possible to actually extend the accuracy of the resummed calculation to the NNLL level which has already been achieved for the p_T variable [19] and this is an avenue for future development. In any case our current prediction matched to fixed-order estimates from MCFM should already enable accurate phenomenological investigation alongside forthcoming Tevatron data [1]. As part of an article in progress we plan to carry out the matching and a detailed phenomenological study of the a_T distribution which we anticipate will shed more light on issues relevant to physics at the LHC in the near future.

Acknowledgments

We wish to thank the authors of ref. [1] for informing us about their experimental study of the a_T distribution. One of us (R.D.) would like to thank the Università degli Studi di Milano-Bicocca and INFN, Sezione di Milano-Bicocca for generous financial support and kind hospitality during the course of this work.

A Born level result

Here we explicitly compute the leading-order and the real part of the next-to-leading order contribution to the a_T integrated cross section defined in eq. (3.3).

At leading order $\Sigma^{(0)}(a_T)$ is just the Born cross section:

$$\Sigma^{(0)}(a_T) = \int_0^1 dx_1 \int_0^1 dx_2 \, \left[f_q(x_1) f_{\bar{q}}(x_2) + q \leftrightarrow \bar{q}\right] \times \\ \times \int d\Phi(l_1, l_2)\, \mathcal{M}^2_{\rm DY}(l_1, l_2)\, \delta\left(M^2 - 2\, l_1.l_2\right) \,, \tag{A.1}$$

where $\mathcal{M}^2_{\rm DY}$ is the Born matrix element reported in eq. (3.1). To this end we look at the Lorentz-invariant phase-space which can be written as

$$\int d\Phi(l_1, l_2) = \frac{1}{2\hat{s}} \int \frac{d^3 l_1}{2(2\pi)^3 l_{10}} \frac{d^3 l_2}{2(2\pi)^3 l_{20}} (2\pi)^4 \delta^4(p_1 + p_2 - l_1 - l_2)\,, \tag{A.2}$$

where we included in addition to the usual two-body phase space a delta function corresponding to holding the invariant mass of the lepton-pair at M^2 and $\hat{s}$ is the partonic centre of mass energy squared $\hat{s} = s\, x_1 x_2$.

We parameterise the four vectors of the incoming partons and outgoing leptons as below (in the lab frame)

$$p_1 = \frac{\sqrt{s}}{2} x_1 \,(1, 0, 0, 1),$$
$$p_2 = \frac{\sqrt{s}}{2} x_2 \,(1, 0, 0, -1),$$
$$l_1 = l_T \,(\cosh y, 1, 0, \sinh y)\,, \tag{A.3}$$

with l_2 being fixed by the momentum conserving delta function.

In the above $\sqrt{s}$ denotes the centre of mass energy of the incoming hadrons while x_1 and x_2 are momentum fractions carried by partons p_1 and p_2 of the parent hadron momenta, while l_T and y are the transverse momentum and rapidity of the lepton with respect to the beam axis and we work in the limit of vanishing lepton masses. In these terms we can express eq. (A.2) as (after integrating over l_2 using the momentum conserving delta function)

$$\int d\Phi(l_1, l_2) = \frac{1}{2\hat{s}} \int \frac{l_T dl_T dy}{4\pi} \delta\left((p_1 + p_2 - l_1)^2\right)\,, \tag{A.4}$$

where we have carried out an irrelevant integration over lepton azimuth. Note that the factor $\delta\left((p_1 + p_2 - l_1)^2\right)$ arises from the vanishing invariant mass of lepton l_2.

In order to obtain the full Born result we need to fold the above phase space with the parton distribution functions[8] and the squared matrix element for the Drell-Yan process to finally obtain

$$\Sigma^{(0)}(M^2) = \int_0^1 dx_1\, f(x_1) \int_0^1 dx_2\, f(x_2)\, \times$$
$$\times \frac{1}{2\hat{s}} \int \frac{l_T\, dl_T\, dy}{4\pi} \delta\left(s\, x_1 x_2 + t_1 + t_2\right) \delta\left(M^2 - x_1 x_2 s\right) \mathcal{M}^2_{\mathrm{DY}}. \tag{A.5}$$

where we used $(p_1 + p_2 - l_1)^2 = s x_1 x_2 + t_1 + t_2$.

We next evaluate the squared matrix element $\mathcal{M}^2_{\mathrm{DY}}$ in eq. (3.1) in terms of the phase space integration variables:

$$t_1 = -2p_1.l_1 = -\sqrt{s}\, x_1\, l_T\, e^{-y}\,, \qquad t_2 = -\sqrt{s}\, x_2\, l_T\, e^{y}\,. \tag{A.6}$$

Inserting these values of t_1 and t_2 in eq. (A.5) we use the constraint

$$\delta(s\, x_1 x_2 + t_1 + t_2) = \delta\left(s\, x_1 x_2 - \sqrt{s}\, l_T \left(x_2\, e^{y} + x_1\, e^{-y}\right)\right)\,, \tag{A.7}$$

to carry out the integration over l_T which gives

$$\frac{1}{8\pi s} \int_0^1 dx_1\, f(x_1) \int_0^1 dx_2\, f(x_2)\, \delta\left(M^2 - x_1 x_2 s\right) \frac{dy}{(x_2 e^{y} + x_1 e^{-y})^2} \mathcal{M}^2_{\mathrm{DY}}\,, \tag{A.8}$$

[8] In order to avoid excessive notation we do not explicitly indicate the sum over incoming parton flavours which should be understood.

where in evaluating $\mathcal{M}^2_{\rm DY}$ one needs to use $l_T = \sqrt{s}\, x_1 x_2/(x_2 e^y + x_1 e^{-y})$, which yields using (A.6)

$$t_1^2 = x_1^2 e^{-2y} \frac{M^4}{(x_2 e^y + x_1 e^{-y})^2}\,, \qquad \text{(A.9)}$$
$$t_2^2 = x_2^2 e^{2y} \frac{M^4}{(x_2 e^y + x_1 e^{-y})^2}\,.$$

Using the above to evaluate $\mathcal{M}^2_{\rm DY}$ in (3.1) we obtain

$$\Sigma^{(0)}(M^2) = \frac{\mathcal{G}}{N_c}\frac{M^4}{\pi s}\int_0^1 dx_1\, f(x_1)\int_0^1 dx_2\, f(x_2)\,\delta\left(M^2 - x_1 x_2 s\right)\int dy\,\mathcal{F}(x_1,x_2,y), \quad \text{(A.10)}$$

where we introduced

$$\mathcal{F}\left(x_1,x_2,y\right) = A_l\, A_q\, \frac{x_1^2 e^{-2y} + x_2^2 e^{2y}}{(x_2 e^y + x_1 e^{-y})^4} + B_l\, B_q\, \frac{x_1^2 e^{-2y} - x_2^2 e^{2y}}{(x_2 e^y + x_1 e^{-y})^4}\,. \qquad \text{(A.11)}$$

Integrating the angular function $\mathcal{F}$ over rapidity over the full rapidity range[9] one finds as expected that the parity violating component proportional to $B_l\, B_q$ vanishes and the result is $A_l\, A_q/(3x_1x_2)$. Thus the final result is (using $x_1x_2 = M^2/s$)

$$\Sigma^{(0)}(M^2) = \mathcal{G}\frac{M^2}{3\pi}\frac{A_l\, A_q}{N_c}\int_0^1 dx_1\, f(x_1)\int_0^1 dx_2\, f(x_2)\,\delta\left(M^2 - x_1x_2 s\right). \qquad \text{(A.12)}$$

We now compute the LO QCD result by considering the emission of a gluon in the Drell-Yan (QCD annihilation) process as well as the contribution of the quark-gluon (QCD Compton) scattering process. Thus we consider the reaction $p_1 + p_2 = l_1 + l_2 + k$ where k is the emitted gluon in the Drell-Yan process and a quark/anti-quark for the Compton process.

The squared matrix elements at this order for the annihilation and Compton processes are reported in appendix B and we shall use those results in what follows below.

We explicitly parameterise the momentum k as below

$$k = k_t\left(\cosh y_k, \cos\phi, \sin\phi, \sinh y_k\right). \qquad \text{(A.13)}$$

The parameterisation of the other particles four-momenta is as in the Born case eq. (A.3). One now has to integrate the squared matrix elements over a three body final state and to this end we introduce the usual Mandelstam invariants

$$\hat{u} = -2p_1.k = -\sqrt{s}\, x_1\, k_t\, e^{-y_k}\,, \qquad \hat{t} = -2p_2.k = -\sqrt{s}\, x_2\, k_t\, e^{y_k}\,. \qquad \text{(A.14)}$$

The Lorentz invariant phase space is now

$$\int d\Phi(l_1,l_2,k) = \frac{1}{2\hat{s}}\int \frac{d^3l_1}{2(2\pi)^3 l_{10}}\frac{d^3l_2}{2(2\pi)^3 l_{20}}\frac{d^3k}{2(2\pi)^3 k_0}(2\pi)^4\delta^4(p_1+p_2-l_1-l_2-k)\,. \quad \text{(A.15)}$$

[9]We can straightforwardly adapt the calculation to include the experimental acceptance cuts when available.

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Following the same procedure as in the Born case we perform the trivial integration over l_2 and obtain the leading order QCD correction to the Born result (A.5) (for the moment we are considering just real emission terms indicated below by the label r)

$$\Sigma_r^{(1)}(M^2) = \sum_{i=A,C} \int_0^1 dx_1\, f(x_1) \int_0^1 dx_2\, f(x_2) \times \\ \times \frac{1}{2\hat{s}} \int \frac{l_T dl_T dy}{4\pi} \frac{d^3k}{2(2\pi)^3 k_0} \delta\left(M^2 + t_1 + t_2 + 2l_1.k\right) \times \\ \times \delta\left(M^2 - x_1 x_2 s \left(1 - \frac{2p_1.k}{\hat{s}} - \frac{2p_2.k}{\hat{s}}\right)\right) \mathcal{M}_i^2 . \tag{A.16}$$

Here the factor $\left(1 - \frac{2p_1.k}{\hat{s}} - \frac{2p_2.k}{\hat{s}}\right)$ accounts for the energy-momentum carried off by the radiated parton k while the index $i = A$ pertains to the QCD annihilation process while $i = C$ indicates the QCD Compton process. Noting that one has as before $t_1 = -\sqrt{s}\, x_1 l_T e^{-y}, t_2 = -\sqrt{s}\, x_2 l_T e^{y}$ and additionally $2\, l_1.k = 2\, l_T k_t (\cosh(y - y_k) - \cos\phi)$, we can use the constraint $\delta\left(M^2 + t_1 + t_2 + 2l_1.k\right)$ to integrate over l_T and the value of l_T (and hence t_1, t_2) is thus fixed in terms of other parameters:

$$l_T = \frac{M^2}{\sqrt{s}\left(x_1 e^{-y} + x_2 e^{y} - 2\frac{k_t}{\sqrt{s}}(\cosh(y - y_k) - \cos\phi)\right)}, \\ t_1 = -\frac{x_1 e^{-y}\, M^2}{\left(x_1 e^{-y} + x_2 e^{y} - 2\frac{k_t}{\sqrt{s}}(\cosh(y - y_k) - \cos\phi)\right)}, \tag{A.17}$$

with the expression for t_2 the same as that for t_1 except that $x_1\, e^{-y}$ in the numerator of the above expression for t_1 is to be replaced by $x_2\, e^{y}$. After integrating away the l_T one gets

$$\Sigma_r^{(1)}(M^2) = \sum_{i=A,C} \int_0^1 dx_1\, f(x_1) \int_0^1 dx_2\, f(x_2) \frac{1}{8\pi s} \int dy \frac{d^3k}{2(2\pi)^3 k_0} \times \tag{A.18} \\ \times \frac{M^2}{\hat{s}\left(x_1 e^{-y} + x_2 e^{y} - \frac{2k_t}{\sqrt{s}}(\cosh(y - y_k) - \cos\phi)\right)^2} \mathcal{M}_i^2\, \delta\left(M^2 - z\, x_1\, x_2 s\right) .$$

where we introduced $z = 1 - 2(p_1.k)/\hat{s} - 2(p_2.k)/\hat{s}$.

We are now ready to integrate over the parton and lepton phase space variables. Since we are interested in the specific cross-section in eq. (3.3), we need to integrate over the phase space such that the value of the a_T is below some fixed value. Further we are interested in the small a_T logarithmic terms so that we consider the region $a_T/M \ll 1$.

To avoid having to explicitly invoke virtual corrections we shall calculate the cross-section for all events *above* a_T and subtract this from the total $\mathcal{O}(\alpha_s)$ result $\Sigma^{(1)}(M^2)$ which can be taken from the literature [36]:

$$\Sigma^{(1)}(a_T, M^2) = \Sigma^{(1)}(M^2) - \Sigma_c^{(1)}(a_T, M^2), \tag{A.19}$$

where we shall calculate $\Sigma_c^{(1)}(a_T, M^2) = \int_{a_T} \frac{d\sigma}{da'_T dM^2} da'_T$.

Moreover since we are interested in just the soft and/or collinear logarithmic behaviour we can use the form of the a_T in the soft/collinear limit derived in the previous section. Thus we evaluate the integrals in eq. (A.18) with the constraint $\Theta\left(k_t|\sin\phi| - a_T\right)$. In order to carry out the integration let us express the parton phase space in terms of rapidity y_k, k_t and ϕ. Thus we have

$$\begin{aligned}\int \frac{d^3k}{2(2\pi)^3 k_0} &= \int \frac{k_t dk_t dy_k d\phi}{2(2\pi)^3} \\ &= \left(\frac{M^2}{16\pi^2}\right)\int\frac{d\phi}{2\pi}\int dy_k \int \frac{dz\, d\Delta}{\sqrt{(1-z)^2-4z\Delta}}\left[\delta(y_k-y_+)+\delta(y_k-y_-)\right],\end{aligned} \tag{A.20}$$

where we used

$$z = 1 - \frac{k_t}{\sqrt{s}x_2}e^{-y_k} - \frac{k_t}{\sqrt{s}x_1}e^{y_k}, \tag{A.21}$$

which follows from the definition of z and where we also introduced the dimensionless variable $\Delta = k_t^2/M^2$. A fixed value of z corresponds to two values of the emitted gluon rapidity

$$y_\pm = \ln\left[\frac{\sqrt{s}\,x_1}{2\,k_t}\left((1-z)\pm\sqrt{(1-z)^2-4z\Delta}\right)\right]. \tag{A.22}$$

The requirement that the argument of the square root in eq. (A.22) be positive sets an upper bound for the z integration, specifically $z < 1 - 2(\sqrt{\Delta+\Delta^2}-\Delta)$. In the limit $\Delta \to 0$, up to corrections of order Δ, this bound reduces to $z < 1 - 2\sqrt{\Delta}$. Having obtained the phase space in terms of convenient variables we need to write the squared matrix elements in terms of the same. We first analyse the QCD annihilation correction and next the Compton piece. In the annihilation contribution one has singularities due to the vanishing of the invariants $\hat{t}$ and $\hat{u}$ with the $1/(\hat{t}\hat{u})$ piece contributing up to double logarithms due to soft and collinear radiation by either incoming parton and the $1/\hat{t}$ and $1/\hat{u}$ singularities generating single logarithms. The double logarithms arise from low energy and large rapidity emissions (soft and collinear emissions) while the single-logarithms from energetic collinear emissions, hence it is the small k_t limit of the squared matrix elements that generates the relevant logarithmic behaviour. Thus we write the squared matrix element $\mathcal{M}_A^2$ in eq. (B.1) in terms of the variables Δ and z and then find the leading small Δ behaviour. Specifically in the $\Delta \to 0$ limit, considering only the $1/\hat{t}$ singular piece, the factor appearing in (A.18) has the following behaviour (keeping for now only the $A_l A_q$ piece of the matrix element):

$$\frac{1}{\hat{s}}\mathcal{M}_A^2 \frac{M^2}{s\left(x_1e^{-y}+x_2e^{y}-\frac{2k_t}{\sqrt{s}}\left(\cosh(y-y_k)-\cos\phi\right)\right)^2} \approx \\ 16\,g^2\,\mathcal{G}\,\frac{A_l\,A_q}{N_c}\,\frac{C_F}{\Delta}\,(1+z^2)\,\frac{M^2}{s}\,\frac{x_1^2\,e^{-2y}+x_2^2\,z^2\,e^{2y}}{(x_1\,e^{-y}+x_2\,z\,e^{y})^4}. \tag{A.23}$$

Performing the integral over all rapidities y of the lepton, the above factor produces $16g^2\mathcal{G}\frac{A_lA_q}{N_c}\frac{C_F}{\Delta}\frac{1+z^2}{3}$. The corresponding B_lB_q piece of the squared matrix element vanishes upon integration over all rapidities.

Since the $1/\hat{u}$ singular term, after integration over all lepton rapidities, gives us the same result as that arising from eq. (A.23), we can write for the annihilation process (using eqs. (A.18), (A.20)) and $g^2 = 4\pi\alpha_s$

$$\Sigma^{(1)}_{A,c} = \mathcal{G}\frac{A_l A_q}{N_c}\int dx_1 dx_2 f(x_1) f(x_2)\frac{M^2}{3\pi}\int_0^1 \frac{d\Delta}{\Delta}\int_0^{1-2\sqrt{\Delta}} dz\ \delta\left(M^2 - x_1 x_2 z\, s\right)$$
$$\int_0^{2\pi}\frac{d\phi}{2\pi}\, C_F\frac{\alpha_s}{2\pi}\frac{2\left(1+z^2\right)}{\sqrt{(1-z)^2 - 4z\Delta}}\Theta\left(\sqrt{\Delta}|\sin\phi| - \frac{a_T}{M}\right). \tag{A.24}$$

Following the same procedure for the Compton process one finds instead

$$\frac{1}{\hat{s}}\mathcal{M}^2_C \frac{M^2}{s\left(x_1 e^{-y} + x_2 e^{y} - \frac{2k_t}{\sqrt{s}}\left(\cosh(y-y_k) - \cos\phi\right)\right)^2}$$
$$\approx 16\, g^2\, \mathcal{G}\,\frac{A_l A_q}{N_c}\frac{T_R}{\Delta}(1-z)\left[z^2 + (1-z)^2\right]\frac{M^2}{s}\frac{x_1^2\, e^{-2y} + x_2^2\, z^2\, e^{2y}}{\left(x_1\, e^{-y} + x_2\, z\, e^{y}\right)^4}, \tag{A.25}$$

which after integration over the rapidity y reduces to $16 g^2 \mathcal{G}\frac{A_l A_q}{N_c}\frac{T_R}{\Delta}(1-z)\frac{z^2+(1-z)^2}{3}$. Thus we have for this piece

$$\Sigma^{(1)}_{C,c} = \mathcal{G}\frac{A_l A_q}{N_c}\int dx_1 dx_2 f(x_1) f(x_2)\frac{M^2}{3\pi}\int_0^1 \frac{d\Delta}{\Delta}\int_0^{1-2\sqrt{\Delta}} dz\ \delta\left(M^2 - x_1 x_2 z\, s\right)$$
$$\int_0^{2\pi}\frac{d\phi}{2\pi}\, T_R\frac{\alpha_s}{2\pi}\frac{(1+z)\left[z^2 + (1-z^2)\right]}{\sqrt{(1-z)^2 - 4z\Delta}}\Theta\left(\sqrt{\Delta}|\sin\phi| - \frac{a_T}{M}\right). \tag{A.26}$$

B Leading order cross section in the small Δ limit

The matrix element squared for the QCD annihilation process from ref. [23] is (in four dimensions)

$$\mathcal{M}^2_A(l_1, l_2, k) = -16\, g^2\, \mathcal{G}\,\frac{C_F}{N_c} M^2 \times$$
$$\times\left\{A_l A_q\left[\left(1 + \frac{\hat{s} - 2t_1 - M^2}{\hat{t}} - \frac{t_1^2 + t_2^2 + \hat{s}\left(t_1 + t_2 + M^2\right)}{\hat{t}\hat{u}}\right) + \left(\hat{u}\leftrightarrow\hat{t}, t_1 \leftrightarrow t_2\right)\right]\right.$$
$$\left. + B_l B_q\left[\left(\frac{\left(\hat{s} + 2t_2 + M^2\right)}{\hat{t}} + \frac{M^2\left(t_1 - t_2\right)}{\hat{t}\hat{u}}\right) - \left(\hat{u}\leftrightarrow\hat{t}, t_1\leftrightarrow t_2\right)\right]\right\}, \tag{B.1}$$

while for the QCD Compton process, if p_2 represents an incoming gluon, one has

$$\mathcal{M}^2_C(l_1, l_2, k) = -16\, g^2\, \mathcal{G}\,\frac{T_R}{N_c} M^2 \times$$
$$\times\left\{A_l A_q\left[\frac{\hat{t} - 2\left(t_1 + M^2\right)}{\hat{s}} + \frac{\hat{s} + 2(t_1 + t_2)}{\hat{t}} + \frac{2}{\hat{s}\hat{t}}\left(\left(t_1 + t_2 + M^2\right)^2 + t_1^2 - t_2 M^2\right)\right]\right.$$
$$\left. + B_l B_q\left[\frac{2\left(t_1 + M^2\right) - \hat{t}}{\hat{s}} + \frac{\hat{s} + 2\left(t_1 + t_2\right)}{\hat{t}} - \frac{2M^2\left(2t_1 + t_2 + M^2\right)}{\hat{s}\hat{t}}\right]\right\}, \tag{B.2}$$

where we corrected small errors (after an independent recomputation of the above) of an apparent typographical nature in the $B_l B_q$ piece of the annihilation result.

Here we report also the electroweak coefficient constants $\mathcal{G}$, A_l, A_q, B_l, B_q for the case of Z boson exchange:

$$\begin{aligned}\mathcal{G}(\alpha, \theta_W, M^2, M_Z^2) &= \frac{4\pi^2\alpha^2}{\sin^4\theta_W \cos^4\theta_W} \frac{1}{(M^2 - M_Z^2)^2 + (\Gamma_Z M_Z)^2}\,, \\ A_f &= a_f^2 + b_f^2\,, \qquad B_f = 2a_f b_f\,, \qquad (f = l, q)\,.\end{aligned} \tag{B.3}$$

All these quantities have been taken from ref. [23], where the reader can find analogous expressions for the case in which a virtual photon is exchanged as well. Following the conventions of ref. [23], we also have

$$\begin{aligned} a_l &= -\frac{1}{4} + \sin^2\theta_W\,, & b_l &= \frac{1}{4}\,, \\ a_{u,c} &= \frac{1}{4} - \frac{2}{3}\sin^2\theta_W\,, & b_{u,c} &= -\frac{1}{4}\,, \\ a_{d,s,b} &= -\frac{1}{4} + \frac{1}{3}\sin^2\theta_W\,, & b_{d,s,b} &= \frac{1}{4}\,. \end{aligned} \tag{B.4}$$

The kinematical variables $\hat{u}$ and $\hat{t}$ are defined in eq. (A.14). In the limit of small Δ both matrix elements become collinear singular. In the annihilation subprocess this occurs when the emitted gluon k is collinear to either p_1 (corresponding to $\hat{u} \to 0$) or p_2 ($\hat{t} \to 0$). The singularity for $\hat{u} \to 0$ occurs at positive gluon rapidity y_k, correspondingly the one for $\hat{t} \to 0$ occurs at negative y_k. The matrix element for the Compton process shows only a collinear divergence when an outgoing quark is collinear to the incoming gluon, corresponding to $\hat{t} \to 0$. In the following we compute the approximated expression of $\mathcal{M}_A^2$ and $\mathcal{M}_C^2$ in the collinear limit $\hat{t} \to 0$. The remaining collinear limit $\hat{u} \to 0$ of $\mathcal{M}_A^2$ gives an identical result after integration over the lepton rapidity.

Neglecting terms of relative order k_t one has

$$\begin{aligned} l_T &\simeq \frac{M^2}{\sqrt{s}\,(x_1 e^{-y} + z x_2 e^{y})}\,, & \hat{t} &\simeq -\frac{k_t^2}{1-z}\,, & \hat{u} &\simeq -\frac{1-z}{z} M^2\,, \\ t_1 &\simeq -\frac{x_1 e^{-y} M^2}{(x_1 e^{-y} + z x_2 e^{y})}\,, & t_2 &\simeq -\frac{x_2 e^{y} M^2}{(x_1 e^{-y} + z x_2 e^{y})}\,, & M^2 &\simeq -(t_1 + z\, t_2)\,. \end{aligned} \tag{B.5}$$

Substituting these expressions in eq. (B.1) and eq. (B.2) one obtains

$$\mathcal{M}_A^2(l_1, l_2, k) \simeq \frac{16\, g^2}{z\, k_t^2}\, \mathcal{G}\, \frac{C_F}{N_c}\, (1+z^2)\, \left[A_l A_q (t_1^2 + z^2 t_2^2) + B_l B_q (t_1^2 - z^2 t_2^2)\right]\,, \tag{B.6}$$

and

$$\mathcal{M}_C^2(l_1, l_2, k) \simeq \frac{16\, g^2}{z\, k_t^2}\, \mathcal{G}\, \frac{T_R}{N_c}\, (1-z)\, [z^2 + (1-z)^2]\, \left[A_l A_q (t_1^2 + z^2 t_2^2) + B_l B_q (t_1^2 - z^2 t_2^2)\right]\,. \tag{B.7}$$

where the collinear singularity $1/k_t^2$ has been isolated. Note that the result is proportional to the Born matrix element in eq. (3.1) with x_2 replaced by $z\, x_2$, indicating that the momentum fraction of the parton entering the hard scattering has been reduced by a factor z after the emission of a collinear gluon.

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